# Effect of Thermal-Softening in Rod Impact Test for the Determination of Dynamic Material Properties of Polycarbonate

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A theory was developed to investigate the effect of thermal-softening in rod impact test for the determination of the dynamic material properties of Polycarbonate, on the basis of onedimensional shock wave propagation phenomena. High velocity rod impact test was performed with flat-ended cylindrical rod specimens. From the geometrical measurements of deformed rod, dynamic material properties were determined by both previous theories and the theory suggested in this work. The variation of temperature rise due to adiabatic plastic deformation with impact velocities and the effect of thermal-softening on the dynamic yield stress were analyzed.

Key Words: Rod Impact Test, Dynamic Material Properties, High-Strain-Rate, Thermal-Softening, Polycarbonate

# 1. Introduction

Polymers are being increasingly used in engineering applications where their behavior under the impact loading is of primary importance. In order to analyze accurately the deformation behavior and develop polymers having high resistance against the impact loading, characteristics and behaviors of engineering polymers under high-strain-rate should be thoroughly investigated by using appropriate testing techniques.

The testing techniques at high-strain-rate are well reviewed in numerous literatures. Especially for the compression testing, drop-weight test, Hopkinson pressure bar test, and rod impact test have been used(Bitans and Whitton, 1972; Holzer, 1979; Staker et al., 1985).

Rod impact test is one of the simplest methods that can provide conditions of high-strain and high-strain-rate which cannot be obtained by other experimental techniques. In rod impact test

the geometrical features of flat-ended cylindrical rod before and after the impact on flat rigid anvil are measured for the determination of dynamic material properties. The dynamic material properties have been calculated through the analyses based on one-dimensional momentum and/or energy conservation(Taylor, 1948; Hawkyard, 1 969; Gillis et al., 1987a, 1987b, 1989; Min et al., 1993) for metals, one-dimensional shock wave propagation(Lee and Tupper, 1954; Hutchings and O'Brien, 1981; Hutchings, 1979; Date, 1984; Min et al., 1992) for metals and polymers, onedimensional elasto-plastic wave propagation(Date, 1982, 1990a, 1992) for metals, and twodimensional dynamic nonlinear finite difference/ element code(Wilkins and Guinan, 1973; Gust, 1982 ; Johnson and Cook, 1983 ; Zerilli and Armstrong, 1987; Johnson and Holmquist, 1988) for metals.

The previous analyses based on onedimensional shock wave propagation for polymers have used one geometrical feature which is final total deformed length, with an additional constant parameter such as critical impact velocity(Hutchings, 1979) or elastic modulus(Date, 1984), which resulted in almost constant dynamic yield stress or the increase of dynamic yield stress with the increase of impact

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velocity. However the temperature rise produced by adiabatic plastic deformation will result in the thermal-softening which affects the dynamic material properties of engineering polymers(Ward, 1983) and results in the decrease of dynamic yield stress with the increase of impact velocity in rod impact test. These are due to their relative sensitivity to the temperature change.

The effects of thermal-softening on the dynamic material properties of engineering polymers have been investigated by the direct measurement of temperature rise through thermocouple(Chou et al., 1973; Hayashi and Yamamura, 1980) and heat-sensitive film(Swallowe et al., 1986; Dowson et al., 1991) for Hopkinson bar test and drop tower test. For rod impact test, although the indirect calculation of temperature rise was performed through the shape memory effect of polymers(Date, 1990b), the variation of temperature rise with impact velocity and the effect of thermal-softening on the dynamic yield stress was not reported.

In this paper a theory was developed to investigate the effect of thermal-softening in rod impact test for the determination of the dynamic material properties of Polycarbonate which is one of engineering polymers, on the basis of one-dimensional shock wave propagation phenomena. The geometrical features of flat-ended cylindrical rod before and after the impact on rigid flat anvil were fully used without any additional constant parameters. And rod impact test was performed with a compressed-air system for the acceleration of Polycarbonate flat-ended cylindrical rod specimens which have the aspect-ratio(ratio of length to diameter) of 4 and 5, in the impact velocity from 150 to 250m/sec. From the geometrical measurements of deformed rod, dynamic material properties were determined from preveious theories and present theory. Finally the variation of temperature rise due to adiabatic plastic deformation with impact velocity and the effect of thermal-softening on the dynamic yield stress were analyzed.

# 2. Theory

# 2.1 Review of previous analyses based on shock wave propagation

For the analyses based on momentum and/or energy conservation, the material has been assumed to be rigid-perfectly plastic material. And Lee and Tupper(1954) proposed an analysis which is based on shock wave propagation, in order to investigate the deformation behavior of high-strengh steel which was assumed to be elastic-plastic material. They found good agreement between their results and predictions based on momentum conservation. Since the maximum elastic strain energy which can be stored in the test specimen is small compared with the plastic work, the theory based on momentum conservation is expected to provide a satisfactory approximation.

None of theories based on momentum and/or energy conservation, however, are suitable for the determination of dynamic material properties of engineering polymers, since any theory must take account of the substantial elastic strain which polymers can suffer before yielding plastically. In the case of polymers, elastic strain will not in general be negligible compared with plastic strain.

Hence Hutchings(1979) proposed an analysis based on one-dimensional shock wave propagation, where it was assumed that the deformation stops when the interaction between elastic wave and plastic wave occurs. The parameters used for the determination of dynamic material properties are impact velocity, total final deformed length, and critical impact velocity which is the impact velocity for the onset of plastic deformation. The critical impact velocity was determined from the relation between fractional reduction in length and impact velocity. This theory has been applied to the determination of dynamic material properties of various engineering polymers(Kukureka and Hutchings, 1981).

Meanwhile Date(1984) performed rod impact test for the determination of dynamic material properties of polyvinylchloride in which rod

Thoery	Hutchings	Date	
Input Variables	V, ρ, k		
	V <sub>c</sub>	E	
Basic Equations	$e = (8e_y^2 + 4ke_y - k^2)/8e_y + \{(k^2 - 8e_y^2 - 4ke_y)^2 - 16e_y(4e_y^3 + 4ke_y^2 + k^2e_y - k^2)\}^{1/2}/8e_y + \{(k^2 - 8e_y^2 - 4ke_y)^2 - 16e_y(4e_y^3 + 4ke_y^2 + k^2e_y - k^2)\}^{1/2}/8e_y + \{(k^2 - 8e_y^2 - 4ke_y)^2 - 16e_y(4e_y^3 + 4ke_y^2 + k^2e_y - k^2)\}^{1/2}/8e_y + \{(k^2 - 8e_y^2 - 4ke_y)^2 - 16e_y(4e_y^3 + 4ke_y^2 + k^2e_y - k^2)\}^{1/2}/8e_y + \{(k^2 - 8e_y^2 - 4ke_y)^2 - 16e_y(4e_y^3 + 4ke_y^2 + k^2e_y - k^2)\}^{1/2}/8e_y + \{(k^2 - 8e_y^2 - 4ke_y)^2 - 16e_y(4e_y^3 + 4ke_y^2 + k^2e_y - k^2)\}^{1/2}/8e_y + \{(k^2 - 8e_y^2 - 4ke_y)^2 - 16e_y(4e_y^3 + 4ke_y^2 + k^2e_y - k^2)\}^{1/2}/8e_y + \{(k^2 - 8e_y^2 - 4ke_y)^2 - (1 - e_y)\} + e_y^2 + e_$		
	$\mathbf{e}_{y} = \frac{\rho \mathbf{V}_{c}^{2} / \mathbf{Y}}{\mathbf{I} + (\rho \mathbf{V}_{c}^{2} \mathbf{Y})}$	$\mathbf{C}_{o} = \left(\frac{\mathbf{E}}{\rho}\right)^{1/2} = \left(\frac{\sigma_{y}}{\rho \mathbf{e}_{y}}\right)^{1/2}$	
Output Variables	Y, e <sub>y</sub> , e		
V : impact velocity	e : nomin	e : nominal strain	
V <sub>c</sub> : critical impact velocity	e <sub>y</sub> : nomin	$e_y$ : nominal yield strain	
$\rho$ : material density	e : strain	e : strain rate	
$\mathbf{k}: 1 - \mathbf{L}_f / \mathbf{L}$	$\sigma_{y}$ : nomi	$\sigma_y$ : nominal yield stress	
L : initial length of rod	Y : dynar	Y : dynamic yield stress	
$L_{f}$ : final total deformed ler	high $C_o:$ elast	$C_o$ : elastic wave velocity	
X : final undeformed length	$\overline{c}_p: \mathbf{C}_p/\mathbf{C}$	$\overline{c}_p: \mathbf{C}_p/\mathbf{C}_o$	
E : elastic modulus	$C_p$ : plast	$C_p$ : plastic wave velocity	

 Table 1
 Comparison between Hutchings' theory and Date's theory

collides with an elastic bar, and observed that contact time is constant even though impact velocity is above the limit impact velocity which is the maximum velocity below which the theory proposed by Hutchings is applicable. From these results, Date assumed the elastic wave velocity is constant and used the constant elastic modulus instead of the critical impact velocity for the determination of dynamic material properties of polymers. The comparison between Hutchigns' theory and Date's theory was described in Table 1.

# 2.2 New theory based on shock wave propagation

Consider a uniform rod of initial dimensions, length L and cross-sectional area A, which impacts a rigid anvil with impact velocity V, as shown in Fig. 1(a). Assuming that V is large enough, a portion of the rod will deform plastically. At time t after the impact, the rod will deform as shown in Fig. 1(b), where h the deformed length, x the undeformed length, and s the displacement of rod. At the end of impact, the rod has the final deformed length H, the final undeformed length X, and the final total deformed length  $L_f$ .

For the development of theory, we use following assumptions which had been used by Hutchings(1979) and Date(1984);



Fig. 1 The process of deformation of flat-ended cylindrical rod impacting rigid anvil; (a) t=0 (b) t=t

- One-dimensional shock wave propagation is considered.
- The deformation of rod takes place at a constant strain rate.
- The stress-strain relationship of a material is the same throughout the duration of impact.
- A linearly-elastic, perfectly-plastic material, where true yield stress is constant, is considered.
- The elastic unloading has the same modulus as that for loading.
- The plastic wave front does stop at the first interaction with elastic wave.

When the impact producing plastic deformation occurs, elastic wave and plastic wave start simultaneously at the interface between impinging rod and anvil, with velocity  $C_o$  and  $C_p$ , respectively. By considering an element of the rod, it may be deduced from the condition of momentum balance that

$$\Delta \sigma = \rho C \Delta v, \tag{1}$$

where  $\rho$  is material density,  $\Delta \sigma$  and  $\Delta v$  are infinitesimal changes in nominal stress and particle velocity, respectively. *C* is the wave velocity in terms of Lagrangian coordinates. The changes in nominal strain  $\Delta e$ , where compressive strain is assumed to be positive, and particle velocity  $\Delta v$ are related by

$$\Delta v = C \Delta e. \tag{2}$$

From Eqs. (1) and (2), the wave velocity is given by

$$C^{2} = (\Delta \sigma / \Delta e) / \rho, \qquad (3)$$

and the elastic wave and plastic wave propagate with velocities given in Eqs. (4) and (5), respectively.

$$C_o = (\sigma_y / \rho e_y)^{1/2}, \tag{4}$$

$$C_{p} = [(\sigma - \sigma_{y}) / \rho(e - e_{y})]^{1/2}, \qquad (5)$$

where  $\sigma_y$  and  $e_y$  are nominal yield stress and nominal yield strain, respectively.

In the plastically deformed region behind a plastic wave front, the nominal stress  $\sigma$  and the permanent plastic strain  $e_p$  can be given as Eqs. (6) and (7), respectively.

$$\sigma = \rho C_p V + \sigma_y [1 - (C_p / C_o)]. \tag{6}$$

$$e_{p} = [1 - (C_{p}/C_{o})^{2}](V - C_{o}e_{y})/C_{p}.$$
 (7)

In Eqs. (6) and (7), V is impact velocity of rod. If we denote  $C_p/C_0$  as  $\overline{C}_p$ , and since the nominal stress  $\sigma(\geq \sigma_y)$  and the dynamic yield (true) stress V are related as Eq. (8),

$$Y = \sigma(1 - e), \tag{8}$$

Eqs. (4)~(7) and  $\overline{C}_p$  become

eр

$$C_o = [Y/(\rho e_y(1-e_y))]^{1/2}, \qquad (9)$$

$$C_{\rho} = [Y/(\rho(1-e)(1-e_{y}))]^{1/2}, \qquad (10)$$
  
$$Y/(1-e) = \rho \bar{C}_{\rho} V$$

$$+Y(1-C_p)/(1-e_y),$$
 (11)

$$= (1 - \bar{C}_{p}^{2})(V - C_{o}e_{y})/C_{p}, \qquad (12)$$

$$\overline{C}_{p} = C_{p}/C_{o} = [e_{y}/(1-e)]^{1/2}.$$
(13)

From Eqs. (9)~(13), the nominal yield strain  $e_y$ , the nominal strain e, and dynamic yield stress Ycan be expressed as function of  $e_p$  and  $\overline{C}_p$ , given by

$$e_{y} = \overline{C}_{p}^{2} (1 - e_{p} - \overline{C}_{p}^{2}) / (1 - \overline{C}_{p}^{4}), \qquad (14)$$

$$e = 1 - (1 - e_p - C_p^2)/(1 - C_p^2), \quad (15)$$

$$(Y/\rho V^2)^{1/2} = [1/\{(1 - e)(1 - e_y)\}]^{1/2}$$

$$[1/(1 - e)$$

$$-(1-\bar{C}_p)/(1-e_y)]^{-1}$$
. (16)

And the critical impact velocity  $V_c$  for the onset of plastic deformation can be determined from the Eqs. (6), (8) and (9) and the conditions for initial plastic deformation, as given by

$$V_{c} = [e_{y}Y/\{\rho(1-e_{y})\}]^{1/2}.$$
 (17)

Since the kinematic conditions such as  $L=H_L$ +X and  $L_f=H+X$  are established, where  $H_L$ is the final deformed length in terms of Lagrangian coordinates, the permanent plastic strain  $e_p$ and the time required for the first interaction of elastic wave and plastic wave  $t_w$  become

$$e_p = -(H - H_L)/H_L,$$
 (18)

$$t_W = H_L/C_P = (2L - H_L)/C_o,$$
 (19)

and the  $e_p$  and  $\overline{C}_p$  can be expressed as a function of geometrical features of deformed rod including final undeformed length X and final total deformed length  $L_f$  as follows

$$e_p = (1 - L_f/L)/(1 - X/L),$$
 (20)

$$\overline{C}_{p} = (1 - X/L)/(1 + X/L).$$
 (21)

Hence if we measure geometrical features of deformed rod with known values of impact veloc-

ity and material density, the dynamic material properties including nominal yield strain, nominal strain, and dynamic yield stress can be determined from Eqs.  $(14) \sim (16)$  without any additional parameters such as constant critical impact velocity used by Hutchings and constant elastic modulus used by Date.

For the strain-rate Hutchings and Date used following equation proposed by Taylor(1948)

$$\dot{e} = V/2(L-X).$$
 (22)

However in the present analysis the strain-rate can be determined from Eqs. (15) and (19) as follows

$$\dot{e} = e/t_{W},\tag{23}$$

where  $\dot{e}$  is also functions of X/L and  $L_f/L$ .

#### 2.3 Error analysis of theories

Relation between nondimensionalized dynamic yield stress  $Y/\rho V^2$  and fractional reduction in length k which is  $1-L_f/L$ , has been plotted in Fig. 2, with variations of nondimensionalized critical impact velocity  $V_c/V$  for Hutching's theory, nondimensionalized elastic modulus  $E/\rho V^2$ for Date's theory, and nondimensionalized final undeformed length X/L for present theory.

Hutchings' theory shows, in Fig. 2(a), that when the impact velocity is near the critical impact velocity the effect of variation of critical impact velocity considerably increases. Since the critical impact velocity is in general determined by the linear extrapolation from the relation of fractional reduction in length and impact velocity, there will be some error in the determination of critical impact velocity. Hence for the reduction of erroneous effect of measured critical impact velocity it seems to be desirable that dynamic material properties are determined from test results having impact velocity near the limit velocity, which satisfies the condition that the plastic wave front must be wholly attenuated at the first interaction with elastic wave.

For Date's theory, it is shown in Fig. 2(b) that the effect of variation of elastic modulus on the dynamic yield stress is negligible at the relatively high value of fractional reduction in length. Even though Date assumed that elastic modulus is



(a) Critical impact velocity  $V_c/V$ 



(b) Elastic modulus  $E/\rho V^2$ 



(c) Final undeformed length X/L



constant without dependency on strain-rate, the elastic modulus depends on the strain-rate and temperature(Daniels, 1989) and hence it seems to be uncertain that the assumption can be applied to rod impact test. Hence for the reduction of erroneous effect of used constant elastic modulus, it seems to be desirable that dynamic material properties are determined from test results having the relatively high value of fractional reduction in length.

Meanwhile present theory shows, in Fig. 2(c), that the effect of variation of final undeformed length on dynamic yield stress diminishes with the increase of fractional reduction in length. Hence for the reduction of erroneous effect of measured final undeformed length, it seems to be desirable that dynamic yield stress is determined from test results having high impact velocity which produces large final total deformed length.

Although the erroneous effects of input paramrters used in the theories decrease with the increase of impact velocity, the dynamic yield stress in the previous theories are dependent on the additional constant parameters which are selected without relation to the deformation behavior of rod. Hence the thermal softening resulting from the temperature rise produced by adiabatic plastic deformation cannot be effectively analyzed.

# 3. Experiment

#### 3.1 Test material

The material of specimen used in the rod impact test is Polycarbonate(PC) which has the physical and thermal properties as shown in Table 2. Rod specimens have been machined to have a flat-ended cylindrical shape of aspectratios(L/D; L and D are length and diameter of rod, respectively) of 4 and 5, where the diameter of rod is 10 mm.

#### 3.2 Experimental setup

Rod impact test was performed using the experimental setup, shown in Fig. 3 at room tempera-

Table 2Physical and thermal properties of polycar-<br/>bonate (Chanda and Roy, 1987)

Density(Mg·m <sup>-3</sup> )	Compressive elastic modulus(GPa)	Specific heat $(cal \cdot g^{-1} \cdot C^{-1})$
1.2	2.4	0.3



Fig. 3 Experimental setup for rod impact test

ture(20°C), which consists of the compressed-air system, the air release system, the accelerating tube, the velocity measurement system, and the rigid flat anvil.

The rod specimens were accelerated by a compressed-air system in the pressure up to 10 MPa, where the required pressure is obtained using air control units including compressor, reservoir, regulator, and accumulator. For the normal impact of rod specimens, the cylindrical accelerating tube was used, which has the length of 1 m and the interior diameter of 10 mm. The compressed-air is released to the accelerating tube by an air release system.

The impact velocity was determined by using the photoelectronic sensors which utilize light beam and time counter(Racal Dana Model 9015/ 11A). The impact velocity can be calculated from the flight time and the distance between light beams. The used range of impact velocity is 150  $\sim 250$  m/sec.

In order to measure the deformed shape of rod specimen after the impact, a recovery box was installed ahead the rigid flat anvil which is the electroslag remelted wrought-steel homogeneous plate having the hardness of HB500. The anvil has been fixed by rigid structure on the ground in order to prevent the movement of the anvil.

After the impact the deformed specimen was thoroughly inspected in order to choose the specimen to be analyzed. When the fracture, the nonaxisymmetrical deformation, and the buckling occur, the specimen was excluded in the analysis. And then the shape of the deformed specimen was enlarged by the profile projector, and the deformed shapes such as final undeformed length, final deformed length, and final total deformed length have been accurately measured by mechanical devices.

#### 3.3 Experimental results

From the geometrical measure of deformed rod, the variation of nondimensionalized final total deformed length  $L_f/L$  and final undeformed length X/L with impact velocity V have been plotted for each aspect-ratio as shown in Fig. 4(a) and (b).  $L_f/L$  decreases with the increase of impact velocity as observed in previous works(Hutchings, 1979; Kukureka and Hutchings, 1981). The X/L decreases with the increase of impact velocity where the degree of decrease is larger than that of  $L_f/L$ .

For the calculation of dynamic material properties using the theory proposed by Hutchings, critical impact velocity  $V_c$  was determined from the relation between the fractional reduction in length k and the impact velocity as shown in Fig. 4(c), where  $V_c$  is determined when k becomes zero and tabulated in Table 3. In the calculation of dynamic material properties using theory proposed by Date, the constant compressive elastic modulus E was used from Table 2.

# 4. Analysis and Discussion

The dynamic yield stress, yield strain, and strain-rate have been determined for Polycarbonate specimens in the impact velocity ranging from 150 m/s to 250 m/s, as shown in Figs. (5)  $\sim$ (7). Calculations were performed by Hutchings' theory which uses final total deformed length and constant critical impact velocity, Date's theory which uses final total deformed length and constant elastic modulus, and the present theory which uses final total deformed length and final

 
 Table 3 Critical impact velocity V<sub>c</sub> for polycarbonate

L/D	$V_c(ms^{-1})$
4	104.7
5	102.3



(a) Nondimensionalized final total deformed length L<sub>c</sub>/L



(b) Nondimensionalized final underformed length X/L



(c) Fractional reduction in length k, with the impact velocity V

Fig. 4 Variation

undeformed length as input parameters.

The previous theories show that the dynamic material properties are nearly constant in the used range of impact velocity. Hutchings' theory results



Fig. 5 Variation of dynamic yield stress Y with impact velocity V for polycarbonate having aspect-ratios

in higher values in dynamic yield stress and lower values in yield strain than Date's theory. For strain rate, the two theories produce same values because Eq. (22) proposed by Taylor has been used. Results by Hutchings' theory are in agreement with previous work(Kukureka and Hutchings, 1981) where rod impact test was performed for Polycarbonate having aspect-ratio 2.5 and it was found that the critical impact velocity is 104 m/s, the dynamic yield stress is 174 MPa, the dynamic yield strain is 0.069, and the strain rate is  $6 \sim 7 \times 10^3$ /sec at room temperature(20°C).

In contrast with the previous theories which show the independency of dynamic material properties on impact velocity, the present theory represents the strong dependency of dynamic material properties on impact velocity. The dynamic yield



Fig. 6 Variation of yield strain e, with impact velocity V for polycarbonate having aspect-ratios

stress decreases and the yield strain increases linearly with the increase of impact velocity, as shown in Figs. 5 and 6, respectively. The strainrate, as shown in Fig. 7, increases linearly with the increase of impact velocity.

These dependency of dynamic material properties on impact velocity is due to the thermalsoftening resulting from the temperature rise produced by adiabatic platic deformation of specimen at high-strain-rate. When a specimen deforms plastically, some of the work done in producing the deformation will appear as heat(Bever et al., 1973; Ward, 1983). Although this is true for all metallic and nonmetallic materials, the temperature rise in polymers during the deformation produces relatively large effects due to their relative sensitivity to the temperature change.

The temperature rise produced by adiabatic

2.20



Fig. 7 Variation of strain-rate e with impact velocity V for polycarbonate having aspect-ratios

plastic deformation can be calculated from the nominal stress and permanent plastic strain ep as given in Eq. (24).

$$\Delta T = \frac{\alpha}{\rho C_v} \int_0^{e_\rho} \sigma de_\rho, \qquad (24)$$

where  $C_v$  is specific heat, and  $\alpha$  is the ratio of the energy dissipated for heat generation to the total work done in producing deformation of specimen. In general  $\alpha$  depends on strain, strain-rate, temperature, and microstructure of material, where the value of  $\alpha$  for metals has been reviewed by Bever et al.(1973). Even though studies on the ratio for polymers, also, have been performed by various workers(Chou et al., 1973; Hayashi and Yamamura, 1980; Date, 1990b), the value of the ratio has relatively broad range from 0.55 to 1.0. Chou et al. tested four commonly used plastics in the strain-rate ranging from  $10^{-4}$ /sec to  $2 \times 10^{3}$ / sec in order to analyze the effect of the tempera-

ture rise. A comparison was made between the computed temperature rise, where all mechanical work done in the deformation of a specimen assumed to be converted adiabatically to heat( $\alpha =$ 1.0), and the measued temperature rise by thermocouple embedded in the specimen. It was concluded that at high-strain and high-strain-rate the measured temperature rise is essentially equal to the value calculated. However Hayashi and Yamamura(1980) showed that the ratio of the measured temperature rise to the calculated temperature rise ranges from 0.4 to 0.7, where the average value is 0.55 for Polycarbonate. From these results they proposed a constitutive equation considering thermal softening for polymers. Meanwhile Date utilized the shape memory effect of polyvinylchloride specimen compressed at various strain-rates in order to deduce the maximum temperature rise of the specimen in rod impact test, and concluded that the temperature of the polyvinylchloride specimen impacted longitudinally at high-strain-rate of 103/sec~104/sec seems to be above the glass transition temperature and the ratio is assumed to be 0.7.

The equation for calulation of temperature rise can be formulated from Eq. (24) by utilizing the dynamic material variables used in present theory, as given by Eq. (25).

$$\Delta T = \frac{\alpha}{\rho C_v} \left[ \frac{Y e_y}{2(1 - e_y)} + Y ln \frac{(1 - e_y)}{(1 - e)} - \frac{Y e_y (1 - e_y)}{2(1 - e_y)^2} \right].$$
 (25)

The first term in parenthesis is elastic loading



Fig. 8 The variation of temperature rise  $\Delta T$  with impact velocity V

work, the second term is plastic loading work, and finally third term is elastic unloading work. The ratio  $\alpha$  has been assumed to be 1.0 because of the ambiguity of the value. The variation of temperature rise with impact velocity has been plotted in Fig. 8 for the aspect-ratio of 4 and 5. The temperature rise increase with the increase of impact velocity, from 22°C to 31°C for both L/D=4 and 5.

In order to analyze the results for the variation of temperature rise with impact velocity, the results have been compared with the previous works which analyzed the effect of temperature on the deformation behavior of Polycarbonate, where the temperature was in the steady-state condition. The variation of the yield stress with strain-rate and temperature for Polycarbonate has been extensively investigated by Rietsch and Bouette(1990) who used the generalized theory given as

$$\frac{Y}{T_{k}} = A_{1} \left[ ln^{2}C_{1}\dot{e} + \frac{Q_{1}}{RT_{k}} \right]$$
$$+ A_{2} \sin h^{-1} \left[ C_{2}\dot{e} \exp \frac{Q_{2}}{RT_{k}} \right], \qquad (26)$$

where R is universal gas constant,  $T_k$  is absolute temperature, and the others are parameters determined by experiments under the constant temperature such as  $A_1 = 6.0 \times 10^{-3} \text{ MPa} \cdot \text{K}^{-1}$ ,  $C_1 = 4.1 \times$  $10^{-30}$  sec,  $Q_1 = 68.0 \text{ kcal} \cdot \text{mol}^{-1}$ ,  $A_2 = 54.0 \times 10^{-3}$ MPa·K<sup>-1</sup>,  $C_2 = 2.9 \times 10^{-7}$  sec, and  $Q_2 = 5.0$  kcal· mol<sup>-1</sup>. Since the variation of dynamic yield stress in Eq. (26) is relatively small in the range of strain-rate determined by present rod impact test, the average strain-rate has been used for the investigation of the relation between dynamic yield stress and temperature where  $\dot{e} = 7.1485 \times 10^3$ /sec for L/D=4 and  $\dot{e}=5.6884\times10^3$ /sec for L/D=5. Kukureka and Hutchings(1981) performed rod impact test in which Polycarbonate rod specimen having aspect-ratio of 2.5 has been heated up to the required temperature before impact and the variation of dynamic yield stress with temperature had been investigated. The experiments showed that the dynamic yield stress decrease with the increase of temperaure.

For the comparison between the effect of transient temperature rise in rod impact specimen and



Fig. 9 The relation between dynamic yield stress and temperature

the effect of constant temperature on the dynamic yield stress, these results have been plotted in Fig. 9, where the temperature has been based on room temperature(20°C). The dynamic yield stress determined from present theory and Eq. (25) decreases from 180 to 140 MPa with the increase of temperature from 42 to 51°C. The dynamic yield stresses determined from the present theory, have relatively higher value than the results obtained by steady-state temperature condition. This is considered to result from the fact that the previous experiments have no consideration about the effect of thermal-softening. And the inclination of decrease of dynamic yield stress with the increase of temperature is relatively severe for present theory. This seems to occur from the assumption of constant  $\alpha$ . The ratio  $\alpha$  is strongly dependent on strain, strain-rate, temperature, and microstructure of material. Hence for the relatively exact determination of temperature rise produced in rod impact test specimen, the more profound investigations of the value of  $\alpha$  at high-strain-rate are required.

# 5. Conclusions

A new theory for determining the dynamic material properties of engineering polymers has been developed on the basis of one-dimensional shock wave propagation phenomena, where the geometrical features of flat-ended cylindrical rod before and after impact on a rigid flat anvil are used. Rod impact test was performed with a compressed-air system to accelerate a Polycarbonate flat-ended cylindrical rod specimens having aspect-ratios of 4 and 5, in the velocity range from 150 to 250 m/sec. The effect of thermal softening due to temperature rise produced by adiabatic plastic deformation in rod impact test, has been reviewed and calculated by using the dynamic material properties obtained for this present theory. From these experiments and analyses, following conclusions have been obtained.

(1) The dynamic material properties of Polycarbonate decreases from 180 Mpa to 140 MPa as the impact velocity increases.

(2) The temperature rise produced in Polycarbonate rod, due to the adiabatic plastic deformation, increases from  $22^{\circ}$ C to  $31^{\circ}$ C as the impact velocity increases.

Additionally, the ratio of the energy, dissipated for heat development, to the total work done in producing deformation of specimen should be investigated for the accurate calculation of the temperture rise in rod impact test for polymers.

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